Look How Little I'm Advertising![*]

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Abstract

This paper studies the role of advertising and prices as signals of quality in a purely static setting, where repeat purchases are suppressed altogether, but where advertising affects demand directly. We first show, under standard regularity assumptions, that the high-quality firm will distort its price upwards and its level of advertising downwards compared to the complete-information case. We then show, under relatively mild additional conditions, that the high-quality firm will choose a level of advertising below that of the low-quality firm, even if the high-quality firm advertises most under complete information. Hence, empirically, a high price and a modest advertising budget may well signal high quality.

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1. Introduction

For several decades, in both the marketing and economics professions, substantial effort has been devoted toward achieving a better understanding of the relationship between prices, advertising expenditures and product quality. A voluminous literature has emerged that includes a variety of empirical and theoretical approaches. Contributions representative of the theoretical approaches and containing discussions of empirical matter include Kirmani & Rao (2000) and Hertzendorf & Overgaard (2001). A survey of the empirical and theoretical studies of advertising is provided by Bagwell (2005).

Beginning with Nelson (1970, 1974), a large proportion of the contributions appearing over the past thirty years have viewed the relationship through a *signaling* lens. The basic idea is that price and/or advertising intensity may serve as signals of unobservable product quality. Some of the main insights can be understood in terms of a *two-phase signaling framework*, in which a monopolist provides a new product of either low or high quality that passes through an introductory phase, in which signaling occurs, and a mature phase, in which consumers are informed of product quality. A key assumption is that the high-quality product has a higher marginal cost of production; for example, a high-quality product may require more expensive raw materials. A high-quality monopolist then finds a lower sales volume relatively more attractive than would a low-quality monopolist. Consequently, in the introductory phase, a high-quality monopolist may signal quality with a *high price* that exceeds the complete-information monopoly price. Likewise, if advertising enhances demand, then a high-quality producer may also signal quality by using a *low level of advertising* that is below the complete-information monopoly advertising level.[1] Once the mature phase is reached, consumers know quality, and so signaling is no longer required. In the mature phase, the high-quality monopolist thus selects its complete-information monopoly price and advertising levels. A low-quality monopolist, by contrast, selects its complete-information monopoly price and advertising levels in both the introductory and mature phases.

The two-phase signaling framework implies that the *price-quality* relationship *weakens* over time, since the high-quality monopolist distorts its price upward (i.e., selects a supra-monopoly price) in the introductory phase and then reduces its price to the complete-information monopoly level in the mature phase. Bagwell & Riordan (1991) establish this prediction in a model in which the monopolist can use only price to signal quality.[2] Overgaard (1991, Ch. 2) shows that the prediction is maintained when adver-

[1] In this framework, if advertising is dissipative, then it is not used as a signal in the introductory period. See Overgaard (1991, Ch. 2) and Bagwell (2005, Section 6.1). We focus here on demand-enhancing advertising.

[2] See Bagwell (1991, 1992) for extensions that allow for endogenous quality, general demand functions and product lines.

tising is also available as a signal.[3] As Overgaard shows, a high-quality monopolist also distorts downward the level of demand-enhancing advertising in the introductory phase. It follows that a high-quality monopolist increases its demand-enhancing advertising as the product moves into the mature phase, and the *advertising-quality* relationship thus *strengthens* over time. An immediate but important implication of the framework is thus that price and advertising move in *opposite* directions over time, when quality is high. Bagwell (2005, Section 6.1) provides further analysis of the two-phase signaling framework and the relationship of its predictions to existing empirical findings.

Consider now the relationship between advertising and quality. Is it positive or negative? In other words, would a high-quality monopolist advertise at a greater or lower level than a low-quality monopolist? And does the relationship between advertising and quality depend on whether the product is in the introductory or mature phase? These are fundamental questions for the literature that analyzes advertising as a signal of quality. In particular, answers to these questions would provide testable predictions as to the sign of the correlation between advertising and quality. It is therefore somewhat surprising that existing work does not offer general answers to these questions.

Upon further reflection, however, it is easy to why such answers are not easily obtained. When demand and marginal cost both rise with quality, it is not immediately clear whether the complete-information monopoly advertising level rises or falls with quality. On the one hand, the assumption that marginal cost rises with quality suggests that the complete-information monopoly advertising level may fall with quality; on the other hand, if the marginal effect of advertising on demand is greater for a high-quality product, then the complete-information monopoly advertising level may rise with quality. Suppose first that the complete-information monopoly advertising level falls with quality. In this case, since the high-quality monopolist distorts advertising downward in the introductory period, it follows immediately that the advertising-quality relationship is negative in the introductory and mature phases. Second, suppose that the complete-information monopoly advertising level rises with quality. The advertising-quality relationship is then positive in the mature phase; however, the sign of the relationship is *a priori* ambiguous in the introductory phase. Intuitively, the high-quality monopolist distorts downward its advertising selection, but this downward distortion is relative to a higher starting point.

In this paper, we make two main contributions. First, we offer further support for the predictions that the price-quality relationship weakens over time while the advertising-quality relationship strengthens over time. Our analysis includes a wide range of demand functions; for example, we study signaling behavior when advertising plays an *informative role* by expanding the size of the market, and also when advertising plays

[3]For related methods, see Bagwell & Ramey (1988), who consider price and advertising as joint signals of cost in a model of limit pricing. See also Zhao (2000).

a *persuasive role* by raising the valuations that consumers have for the product. Second, and most importantly, we find that, for a wide range of demand functions, the relationship between advertising and quality in the introductory phase is *negative*. In other words, whether or not a high-quality monopolist would advertise more than a low-quality monopolist in a complete-information setting, we establish the novel finding that the high-quality monopolist selects a *lower* level of advertising than would a low-quality monopolist in the introductory phase. Our analysis thus offers support for the hypothesis that the advertising-quality correlation is negative in the introductory phase and becomes less negative or even positive in the mature phase.

To develop these findings, we analyze a *static signaling game*. The game has a monopolist who is privately informed of the quality of its product, which is high or low. The monopolist then selects a price and a level of demand-enhancing advertising. Consumers observe the price and advertising levels, form beliefs as to product quality, and then demand the corresponding units from the monopolist. In terms of the two-phase signaling framework described above, the static signaling game fully captures the introductory phase of the product. Since profits in the mature phase are independent of introductory-phase selections, we need not explicitly model the mature phase. Instead, we proceed with the understanding that price and advertising levels would be set at their complete-information levels, once the mature phase is reached. This approach seems reasonable if the introductory and mature phases are regarded as periods in the product's life cycle that are sufficiently far apart. Our approach would be less compelling if we were interested in adjacent periods within a broader introductory phase, since in this case the interaction between price, advertising and repeat-purchase dynamics would be of central importance.[4]

Our work relates to previous empirical and theoretical work. As summarized by Bagwell (2005, Section 3.2.5), empirical analyses of the advertising-quality relationship do not offer strong support for the hypothesis of a systematic, positive relationship between advertising and quality. Work by Tellis and Fornell (1988), however, suggests

[4] Thus, we essentially follow the tradition of the signaling literature by directly assuming away signaling over several periods, when separation of types can be accomplished in one go. Recently, Kaya (2005) has shown how distributing the signal (hence, the costly distortion) over several periods may be cost-effective for the strong (high-quality) type and, thus, associated with least-cost separating equilibria. Technically, the reasons are similar to why a Sender may chose to signal in several dimensions (e.g., price, advertising and warranty) when one (say, price) is enough. Increasing the dimensionality of the signal space (whether temporally or in Euclidean fashion) may effectively weaken the incentive pressure from weak types. Hence, high-dimensional signals will tend to be favored by standard equilibrium-selection techniques. See also Bagwell & Riordan's (1991) analysis of the "hindsighted consumer" model. There, a new consumer uses the monopolist's current and past prices when forming beliefs as to product quality, and in the least-cost separating equilibrium the monopolist may "front load" the signaling process (i.e., select a very high price in the introductory period) in order to enjoy a more profitable price in the later period.

that the advertising-quality relationship strengthens as products mature. This is broadly consistent with the findings reported here. Recent work by Horstmann and MacDonald (2003) is of particular interest. They study the compact disc player market. Using panel data on advertising and pricing during 1983-92 and controlling for product features, firm heterogeneity and aggregate effects, they provide evidence that advertising increases after a disc player is introduced, while price moves in the opposite direction and falls from the start. As Horstmann and MacDonald observe, such a pattern is not easily reconciled with the predictions of models in which dissipative advertising is used to signal quality. This pattern, however, is consistent with the predictions developed here.

Most previous theoretical models assume that advertising is dissipative. For example, Kihlstrom and Riordan (1984) consider a model of dissipative advertising as a signal of quality, when firms are price-takers. Milgrom and Roberts (1986) consider a model in which a monopolist uses price and dissipative advertising to signal quality to consumers. They focus on the repeat-purchase dynamics that arise within the introductory phase and report conditions under which the high-quality monopolist may use dissipative advertising as a signal during the initial portion of this phase. Their model predicts that the advertising-quality relationship is positive and decreases over time. Allowing that advertising is demand-enhancing and focusing on the introductory (signaling) and mature (complete-information) phases, we offer support for precisely the opposite pattern. Finally, Zhao (2000) and Orzach, Overgaard & Tauman (2002) offer some findings that are related to those reported here. In comparison, we develop our findings for a general set of demand functions and without the specific structure that is required to analyze repeat-purchase behavior.

The outline of the paper is as follows. In Section 2, we set up the basic model in general form and present some results for the complete-information benchmark. Section 3 turns to the incomplete-information case, presents the analytical approach for the general case and provides some key propositions. In Sections 4 and 5, we provide further results under the informative and persuasive views of advertising, respectively. Section 6 briefly concludes, while some proofs are collected in the Appendix.

2. Model and Complete-Information Benchmark

We first set up a general model of price and advertising signals of the quality of an experience good in the static case. Potential customers are, initially, incompletely informed about the quality of the good on offer. However, prior to making their purchasing decisions, potential customers observe the price and the advertising intensity of the monopoly seller.[5] These variables, chosen by the seller, can therefore serve as signals

[5]Below, advertising intensity can have several interpretations. In some of the discussion we refer to the advertising expenditures incurred by the seller, while in other places we refer to the number of

which influence the beliefs of potential customers. After beliefs are formed, purchases are made, payoffs are realized, and the static signaling game ends.

In this section, we present our notation and the basic assumptions of our model. We also define and discuss the complete-information benchmark. We postpone the formal definition and analysis of the static signaling game until the next section.

2.1. Notation and basic assumptions

Quality. We assume, for simplicity, that quality, q, may either be low or high. Thus, $q \in \{L, H\}$, where $0 \leq L < H$.

Price. The price set by the firm can take on any non-negative value: $p \geq 0$.

Advertising. The level of advertising expenditures (or the number of ads) chosen by the firm can take on any non-negative value: $A \geq 0$.

Beliefs. Let the consumer assessment of the probability that quality is high (H) after observing some signal, (p, A), be denoted by $b(p, A) \in [0, 1]$. $b^0 \in (0, 1)$ denotes the prior probability with which *Nature* selects $q = H$.

Demand. Consumer demand as a function of price and advertising given some beliefs is denoted $D(p, A; b)$. Demand is assumed to be suitably regular: positive, continuous and decreasing in p, increasing and concave in A, and continuous and increasing in b.[6]

Unit costs. Let unit costs be denoted by $c(q)$, and assume that $c(q) \geq 0$, $q \in \{L, H\}$. The case where unit costs are increasing in quality, that is, $c(H) > c(L)$, will be referred to as the *normal case.*

Payoffs. Firm payoffs (or profits) as a mapping from signals, (p, A), given beliefs and actual type, (b, q), are

$$\pi(p, A; b, q) = (p - c(q))D(p, A; b) - A.$$

We assume that $\pi(p, A; b, q)$ is nicely behaved for any b and q. That is, there is a unique, interior maximizer of $\pi(p, A; b, q)$, which we denote $(p_m(q, b), A_m(q, b))$.

2.2. The complete-information benchmark

In the complete-information benchmark, potential customers know the quality on offer, and the *high-quality* firm simply solves

$$\max_{p,A} \pi(p, A; 1, H),$$

advertising messages. What is important is that potential customers observe these or, at least, can form an informed opinion about them.

[6] Throughout, we say that a function $f(x)$ is *increasing* over some interval $[x_L, x_H]$ if, for all x_1 and x_2 in this interval such that $x_2 > x_1$, we have that $f(x_2) > f(x_1)$. Analogously, $f(x)$ is *decreasing* over this interval if $f(x_2) < f(x_1)$.

which gives rise to the price-advertising pair $(p_m(H,1), A_m(H,1)) \equiv (p_M(H), A_M(H))$ and payoffs $\pi(p_m(H,1), A_m(H,1); 1, H) \equiv \Pi_M(H) > 0$. Similarly, the *low-quality* firm solves

$$\max_{p,A} \pi(p, A; 0, L),$$

and we obtain $(p_m(L,0), A_m(L,0)) \equiv (p_M(L), A_M(L))$ and $\pi(p_m(L,0), A_m(L,0); 0, L) \equiv \Pi_M(L) > 0$.

3. Incomplete Information and Signaling

We turn next to a formal analysis of the static signaling game. We begin by defining our equilibrium concept and presenting some basic characteristics of separating equilibria. We then define and characterize the least-cost separating equilibrium outcome.

3.1. Equilibrium concept

A *Perfect Bayesian Equilibrium* of the static signaling game is a set of strategies, $\{p_q, A_q\}_{q=L,H}$, and beliefs, $b(p, A)$, such that: (i) for each $q \in \{L, H\}$, (p_q, A_q) maximizes $\pi(p, A; b(p, A), q)$, and (ii) $b(p, A)$ is derived from the equilibrium strategies using Bayes' rule whenever possible. In a *separating equilibrium*, $(p_H, A_H) \neq (p_L, A_L)$ and thus $b(p_H, A_H) = 1 > 0 = b(p_L, A_L)$. A *pooling equilibrium* occurs when $(p_H, A_H) = (p_L, A_L)$ and thus $b(p_H, A_H) = b^0$.

In a separating equilibrium, the low-quality firm is revealed and can do no better than to make its complete-information selections, $(p_L, A_L) = (p_M(L), A_M(L))$, and earn the corresponding profit $\Pi_M(L)$. Hence, if the high-quality firm is to separate, it must choose a price-advertising pair, (p_H, A_H), which the low-quality firm has no incentive to mimic, that is,

$$\pi(p_H, A_H; 1, L) \leq \Pi_M(L). \tag{3.1}$$

To make the signaling problem interesting, we assume throughout that this constraint is binding, that is,

$$\pi(p_M(H), A_M(H); 1, L) > \Pi_M(L). \tag{3.2}$$

Thus, the low-quality firm has an incentive to mimic the complete-information selection of the high-quality firm, if this fools potential customers. Hence, to separate, the high-quality firm must distort its selection, (p_H, A_H), away from the complete-information maximizer, $(p_M(H), A_M(H))$.

3.2. Least-cost separating equilibrium

The only equilibrium outcome to survive standard refinement techniques (see, e.g., Cho and Kreps (1987)) is the so-called *least-cost* separating outcome,[7] which entails that the high-quality firm chooses (p_H, A_H) to solve the following problem:

$$\max_{p,A} \pi(p, A; 1, H) \text{ s.t. } \pi(p, A; 1, L) = \Pi_M(L). \qquad (PH)$$

To capture the equilibrium relationship between price, advertising and quality, our first aim is to characterize the solution to *(PH)*. To facilitate this, we build on methods introduced by Bagwell & Ramey (1988) and define the following "extended" payoff function

$$\widehat{\pi}(p, A; c) = (p - c)D(p, A; 1) - A.$$

Heuristically, $\widehat{\pi}(p, A; c)$ is just a standard payoff function of a firm with constant unit costs c, who faces demand $D(p, A; 1)$, charges a price p and incurs advertising expenditures A.[8] Assume that $\widehat{\pi}(p, A; c)$ is well-behaved, in the sense that it has a unique, interior maximizer.[9] Let

$$\varphi(c) \equiv (p(c), A(c)) = \arg\max_{p,A} \widehat{\pi}(p, A; c).$$

This gives rise to the payoffs

$$\widehat{\Pi}(c) \equiv \widehat{\pi}(p(c), A(c); c).$$

Note, in particular, that $\varphi(c(H)) = (p(c(H)), A(c(H))) = (p_M(H), A_M(H))$. Finally, it is convenient to impose the following boundary conditions: there exist $c_b > c(L)$ and $c_s < c(L)$ such that $\max\{\pi(\varphi(c_b); 1, L), \pi(\varphi(c_s); 1, L)\} < \Pi_M(L)$.

Then, we can state the following results (proofs are in the Appendix; see also Overgaard (1991, Chapter 2) and the discussion in Bagwell (2005, Section 6.1)).

Proposition 1. *The solution to (PH) coincides with $\varphi(\widetilde{c})$ for some $\widetilde{c}$.*

Proposition 2. *In the normal case where $c(H) > c(L)$, the solution to (PH) is at $\varphi(\widetilde{c})$ for some $\widetilde{c} > c(H)$.*

[7] In the Appendix, following the proofs of Proposition 1 and 2 below, we show that no pooling profiles survive the Cho-Kreps (1987) refinement.

[8] In other words, $\widehat{\pi}(p, A; c)$ represents the profit function for a firm that selects p and A, faces unit cost c and offers a product that is known to be high quality.

[9] This may put some domain restrictions on c, but this is not important for present purposes.

Thus, in the normal case, the high-quality firm makes a *cost-increasing* distortion, which implies that the high-quality firm behaves as if there is complete information, but it has a higher unit cost $\widetilde{c}$, $\widetilde{c} > c(H)$.

Assumption (*The regular case*): $p(c)$ is continuous and increasing in c, and $A(c)$ is continuous and decreasing in c. Further, there exists some (finite) $\overline{c}$ such that $A(\overline{c}) = A_M(L)$.

What we label the *regular case* is just meant to capture that, with complete information, a high-quality firm would increase price and decrease advertising outlays, if the unit costs of production were to increase. The last part of the assumption ensures that there is some (finite) hypothetical level of unit costs, such that under complete information a high-quality firm with this level of unit costs would advertise less than a low-quality firm. This allows us to state the main result for the general case.

Proposition 3. *In the normal and regular case, the high-quality firm distorts its price upwards and its advertising level downwards in the least-cost separating equilibrium outcome. That is,*

$$p_H = p(\widetilde{c}) > p_M(H) \text{ and } A_H = A(\widetilde{c}) < A_M(H).$$

So, we have shown that the high-quality firm distorts advertising downwards to signal quality under the stated assumptions. At the current level of generality, this is about all we can say. However, we would like to compare the equilibrium choice of the high-quality firm, $A_H = A(\widetilde{c})$, with that of the low-quality firm, $A_L = A_M(L)$. Of course, if costs and demands are such that $A_M(H) \leq A_M(L)$, so that the complete-information level of advertising falls with quality, then the high-quality firm must advertise less than the low-quality firm to signal quality. However, the more interesting case is where the complete-information level of advertising rises with quality: $A_M(H) > A_M(L)$. The key question is whether the high-quality firm will have to lower its advertising intensity to a level *below* the complete-information level of the low-quality firm. In the following, we address this question.

We motivate this question in the Introduction. Here, we make a couple of additional observations. First, at an empirical level, the resolution of this question may be valuable by clarifying the predicted relationships among observable variables. In particular, it is sometimes argued that, according to theory, both price and advertising outlays should be positively related to quality. Quality may be difficult to observe, however, and price and advertising selections may be more readily quantified. Therefore, one might look for a positive relationship between price and advertising as a weak test of the theory. Of course, if it turns out that theory predicts a robust and negative relationship between advertising and quality, then the absence of a positive relationship between price and advertising would not constitute evidence against the theory.

Second, on a more positive level, the primary aim of the following sections is to shed some additional light on the predicted relationship between advertising and quality. In particular, we study how various specifications of the influence of advertising on demand (for fixed consumer perceptions of quality) affect the predicted equilibrium relationship between advertising and quality. By establishing general relationships within a unified framework, we hope to facilitate a more ambitious empirical research strategy *vis a vis* the relationship between price, advertising and the quality of experience goods.

We proceed by considering demand specifications that are evocative of the informative and persuasive roles that advertising may respectively play. Henceforth, we restrict our attention to the normal case; thus, in the following, we maintain the assumption that $c(H) > c(L)$. For each of the demand specifications, we then investigate whether the regular case is obtained and compare the equilibrium advertising levels for high and low quality.

4. The Informative View of Advertising

We now focus on advertising that plays an informative role. We begin with a simple base case and then turn to a generalized version of informative advertising. Throughout the section, our focus is on establishing conditions under which the high-quality firm advertises less than would the low-quality firm in the least-cost separating equilibrium outcome. At the end of the section, we also offer a novel interpretation of informative advertising that centers on the notion that informative advertising can work by activating existing memories in the minds of consumers.

4.1. The base case

To obtain some further results on the relationship between quality and advertising, let us first consider the following class of demands

$$D(p, A; b) = G(A)d(p; b),$$

where $G(\cdot) > 0$ satisfies $G' > 0 > G''$ and $d(\cdot;\cdot) > 0$ satisfies $d_p < 0 < d_b$.[10] This is essentially the second class of demand functions considered in Bagwell (2005, Section 4.1.2), which is taken to reflect the *informative* view of advertising.[11] Under this interpretation, $G(A)$ captures the general manner in which advertising increases the size of the market. Observe that advertising shifts demand horizontally outwards as illustrated in Fig. 1. (The figure is drawn for the case in which d is linear in p.)

[10] Thus, we now add some structure and assume that demand functions are differentiable. This assumption enables us to work with first-order conditions.

[11] For this class of demands, price elasticity is independent of advertising.

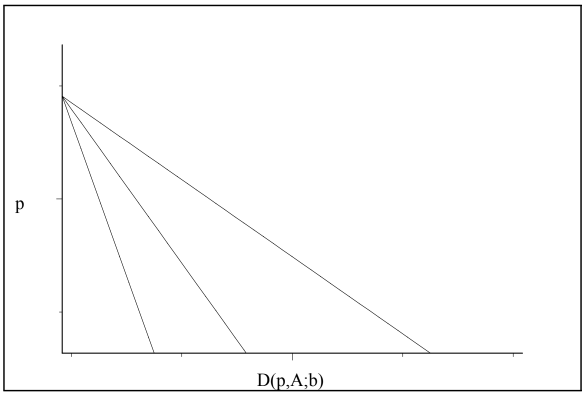

Fig. 1: Informative advertising - Base case

Following Butters (1977), one interpretation of this demand specification is that a greater number of advertising messages increases the number of customers reached from a given population. Consumers reached by advertising messages are said to be informed about the existence of the (new) experience good. Given this interpretation, it makes good sense to assume that $G(A)$ is increasing and concave in A, where A is proportional to the number of messages sent.[12]

The payoff functions defined in the previous section can then be written as

$$\pi(p, A; b, q) = G(A)(p - c(q))d(p; b) - A$$

and

$$\widehat{\pi}(p, A; c) = G(A)(p - c)d(p; 1) - A.$$

The latter has a maximum at $(p(c), A(c))$ which defines

$$\widehat{\Pi}(c) \equiv \widehat{\pi}(p(c), A(c); c) = G(A(c))(p(c) - c)d(p(c); 1) - A(c).$$

Using the first-order conditions, we have

$$\widehat{\pi}_p(p, A; c) = G(A)\{(p - c)d_p(p; 1) + d(p; 1)\} = 0.$$

For an interior solution, this implies that the optimal price is independent of the advertising level, that is,

$$(p - c)d_p(p; 1) + d(p; 1) = 0$$

[12]To be specific, assume that there is a given total mass of homogenous, potential customers, each with individual demands as a function of price and beliefs captured by $d(p; b)$. Assume, in addition, that a fraction $G(A)$ of these potential customers is reached by an advertising campaign of "size" A.

Thus, $p(c)$ solves

$$\max_p (p-c)d(p;1),$$

which immediately implies that $p(c)$ is increasing in c, while $(p(c)-c)d(p(c);1)$ is decreasing in c. In addition, we have

$$\widehat{\pi}_A(p,A;c) = G'(A)(p-c)d(p;1) - 1 = 0$$

into which we can substitute $p(c)$ to obtain

$$G'(A)(p(c)-c)d(p(c);1) - 1 = 0.$$

Given our assumptions that $G_A > 0 > G_{AA}$, since $(p(c)-c)d(p(c);1)$ is decreasing in c, we conclude that the optimal advertising level, $A(c)$, must be decreasing in c. Thus, for this class of demands, we are certainly in the regular case, where $p(c)$ is increasing while $A(c)$ is decreasing in costs.

4.1.1. Comparison of the equilibrium advertising levels

We turn now to the comparison of $A_H = A(\widetilde{c})$ and $A_L = A_M(L)$ for this class of demands. Under our maintained assumption that $c(H) > c(L)$, we know from Proposition 2 that $\widetilde{c} > c(H)$. Having established that we are in the regular case, we also know that $A(\widetilde{c}) < A_M(H)$. Thus, if $A_M(H) \leq A_M(L)$, then we may immediately conclude that $A_H = A(\widetilde{c}) < A_M(H) \leq A_M(L) = A_L$, and so the high-quality firm then advertises less than the low-quality firm, in the least-cost separating equilibrium outcome.

The more interesting case occurs when $A_M(H) > A_M(L)$. Then $A_H = A(\widetilde{c}) < A_M(H)$ and $A_L = A_M(L)$, but it is not immediately clear whether $A_H < A_L$. To analyze this case, we may suppose, hypothetically, that the putative, separating profile is made up of $(p_L, A_L) = (p_M(L), A_M(L))$ and $(p_H, A_H) = (p(c^*), A(c^*)) = \varphi(c^*)$, where $A(c^*) = A_M(L)$. That is, we suppose that both types choose the same advertising level in the putative profile. Notice that since we are in the regular case and $A(c^*) = A_M(L) < A_M(H)$, it follows that $c^* > c(H) > c(L)$. Next, we ask whether the low-quality type would want to pick (mimic) $\varphi(c^*)$, provided that $b = 1$ following $\varphi(c^*)$. If the answer is affirmative, then the high-quality firm will have to distort *further* to some $\varphi(\widetilde{c})$ with $\widetilde{c} > c^*$.[13] That is, it must be the case that $A_H < A_L$ in the least-cost separating equilibrium outcome.

The relevant payoffs can be written as

$$\pi(p(c^*), A(c^*); 1, L) = G(A(c^*))(p(c^*) - c(L))d(p(c^*); 1) - A(c^*)$$

[13] As we confirm in the Appendix, $\pi(\varphi(c); 1, L)$ is decreasing in c for $c > c(L)$.

and

$$\Pi_M(L) = \pi(p_M(L), A_M(L); 0, L) = G(A_M(L))(p_M(L) - c(L))d(p_M(L); 0) - A_M(L).$$

Now, by construction $A(c^*) = A_M(L)$, and we can write the difference between the two payoffs from the perspective of the low-quality firm as

$$\begin{aligned}
& \pi(p(c^*), A(c^*); 1, L) - \Pi_M(L) \\
= \; & G(A(c^*))(p(c^*) - c(L))d(p(c^*); 1) - A(c^*) \\
& -G(A_M(L))(p_M(L) - c(L))d(p_M(L); 0) + A_M(L) \\
= \; & G(A(c^*))[(p(c^*) - c(L))d(p(c^*); 1) - (p_M(L) - c(L))d(p_M(L); 0)],
\end{aligned}$$

which takes the same sign as

$$(p(c^*) - c(L))d(p(c^*); 1) - (p_M(L) - c(L))d(p_M(L); 0).$$

We further note that

$$\widehat{\Pi}(c^*) = G(A(c^*))(p(c^*) - c^*)d(p(c^*); 1) - A(c^*).$$

Using the first-order conditions, it follows that

$$G'(A(c^*))(p(c^*) - c^*)d(p(c^*); 1) - 1 = 0$$

and

$$G'(A_M(L))(p_M(L) - c(L))d(p_M(L); 0) - 1 = 0.$$

But, then $A(c^*) = A_M(L)$ must imply that

$$(p(c^*) - c^*)d(p(c^*); 1) = (p_M(L) - c(L))d(p_M(L); 0).$$

Since $c^* > c(L)$ we also have that

$$(p(c^*) - c(L))d(p(c^*); 1) > (p(c^*) - c^*)d(p(c^*); 1),$$

and we conclude that

$$\begin{aligned}
& (p(c^*) - c(L))d(p(c^*); 1) - (p_M(L) - c(L))d(p_M(L); 0) \\
> \; & (p(c^*) - c^*)d(p(c^*); 1) - (p_M(L) - c(L))d(p_M(L); 0) \\
= \; & 0.
\end{aligned}$$

Thus,

$$\pi(p(c^*), A(c^*); 1, L) > \Pi_M(L),$$

and we see that the low-quality firm would deviate from $(p_M(L), A_M(L)) = (p_M(L), A(c^*))$ to $\varphi(c^*) = (p(c^*), A(c^*))$, provided that $b(\varphi(c^*)) = 1$. Thus, the high-quality firm must change its selection from $\varphi(c^*)$ to some $\varphi(\widetilde{c})$ where $\widetilde{c} > c^*$, in order to separate from the low-quality firm. Hence,

$$A_H = A(\widetilde{c}) < A_M(L),$$

and we conclude that the high-quality firm advertises less than the low-quality firm in the least-cost separating equilibrium for the class of demands considered.

We have thus now established that this conclusion holds irrespective of whether $A_M(H) \leq A_M(L)$ or $A_M(H) > A_M(L)$. Hence, we can summarize our results as follows.

Proposition 4. *Under the informative view of advertising, the high-quality firm advertises less than the low-quality firm in the least-cost separating equilibrium outcome.*

Comment. Somewhat surprisingly, this is the first time a result along these lines has been formally stated in the signaling literature with any degree of generality. Zhao (2000) only considers the very special case where $G(A) = \frac{a}{1+a}$ and $d(p;1) = R - \frac{p}{H}$. Compared to Zhao (2000), the result of this section is very general. It requires only standard regularity conditions on the monopoly-pricing problem *and* multiplicative separability of demand in price and advertising.

4.2. Generalized version of informative advertising

Following the same notation as above, we can consider a more general class of demands

$$D(p, A; b) = G(A; b)d(p; b),$$

where $G(\cdot) > 0$ satisfies $G_A > 0$, $G_b \geq 0$, and $G_{AA} < 0$ while $d(\cdot) > 0$ satisfies $d_p < 0 < d_b$. Note that this class retains the multiplicative separability of demand in price and advertising. In addition, let us assume that $\ln G(A, b)$ has a non-positive cross-partial, that is,

$$\frac{d}{db}\left(\frac{G_A}{G}\right) \leq 0.$$

Under the given assumptions, a sufficient condition for this inequality is $G_{Ab} \leq 0$. This includes the base case above, where $G_b \equiv 0 \equiv G_{Ab}$.

The profit function of the firm can now be written as

$$\pi(p, A; b, q) = (p - c(q))G(A; b)d(p; b) - A.$$

As above, we let $(p_M(L), A_M(L))$ maximize $\pi(p, A; 0, L)$ and $(p_M(H), A_M(H))$ maximize $\pi(p, A;\ 1, H)$. For future reference, we observe that the first-order condition for $A_M(L)$ is given as

$$\pi_A(p_M(L), A; 0, L) = (p_M(L) - c(L))G_A(A; 0)d(p_M(L); 0) - 1 = 0.$$

Similarly, the extended profit function is

$$\widehat{\pi}(p, A; c) = (p - c)G(A; 1)d(p; 1) - A.$$

The first-order conditions for the extended profit function define the maximizer $\varphi(c) \equiv (p(c), A(c))$ and are given by

$$\widehat{\pi}_p(p, A; c) = G(A; 1)\{(p - c)d_p(p; 1) + d(p; 1)\} = 0$$

and

$$\widehat{\pi}_A(p, A; c) = G_A(A; 1)(p - c)d(p; 1) - 1 = 0.$$

Notice that $p(c)$ thus maximizes $(p - c)d(p; 1)$ and satisfies

$$(p - c)d_p(p; 1) + d(p; 1) = 0.$$

As before, this implies that $p(c)$ is increasing in c, while $(p - c)d(p; 1)$ is decreasing in c. Substituting $p(c)$ into the first-order condition for $A(c)$, we thus have that

$$\widehat{\pi}_A(p(c), A; c) = G_A(A; 1)(p(c) - c)d(p(c); 1) - 1 = 0.$$

Given our assumptions that $G_A > 0 > G_{AA}$, since $(p(c) - c)d(p(c); 1)$ is decreasing in c, we have that $A(c)$ is decreasing in c .

We conclude that this model falls within the regular case, in that $p(c)$ is increasing in c while $A(c)$ is decreasing in c. The argument is precisely the same as that above.

4.2.1. Comparison of the equilibrium advertising levels

Given our maintained assumption that $c(H) > c(L)$ and our finding that the model falls within the regular case, we know that $A_H = A(\widetilde{c})$ where $\widetilde{c} > c(H)$ and $A(\widetilde{c}) < A_M(H)$. Thus, as above, if $A_M(H) \leq A_M(L) = A_L$, then we may immediately conclude that the high-quality firm advertises less than the low-quality firm: $A_H < A_L$. The interesting case occurs when $A_M(H) > A_M(L)$. To analyze this case, we again hypothesize a putative separating profile in which $(p_L, A_L) = (p_M(L), A_M(L))$ and $(p_H, A_H) = (p(c^*), A(c^*)) = \varphi(c^*)$, where $c^* > c(H)$ and $A(c^*) = A_M(L)$. We again ask whether the low-quality monopolist would be willing to mimic $\varphi(c^*)$.

We evaluate the difference in profits from the perspective of the low-quality firm

$$\begin{aligned}
& \pi(p(c^*), A(c^*); 1, L) - \pi(p_M(L), A_M(L); 0, L) \\
= \; & G(A(c^*); 1)(p(c^*) - c(L))d(p(c^*); 1) - A(c^*) \\
& -G(A_M(L); 0)(p_M(L) - c(L))d(p_M(L); 0) + A_M(L) \\
= \; & G(A(c^*); 1)(p(c^*) - c(L))d(p(c^*); 1) - G(A_M(L); 0)(p_M(L) - c(L))d(p_M(L); 0) \\
> \; & G(A(c^*); 1)(p(c^*) - c^*)d(p(c^*); 1) - G(A_M(L); 0)(p_M(L) - c(L))d(p_M(L); 0) \\
= \; & \frac{G(A(c^*); 1)}{G_A(A(c^*); 1)} - \frac{G(A(c^*); 0)}{G_A(A(c^*); 0)} \\
\geq \; & 0,
\end{aligned}$$

where the second equality uses $A(c^*) = A_M(L)$, the first inequality follows since $c^* > c(H) > c(L)$, the following equality uses the first-order conditions for $A(c^*)$ and $A_M(L)$ along with $A(c^*) = A_M(L)$, while the final inequality uses our assumption on the cross-partial of $\ln G(A; b)$.

Thus, the low-quality firm has an incentive to deviate from $(p_M(L), A_M(L))$ to $\varphi(c^*)$, provided that $b(\varphi(c^*)) = 1$. The high-quality firm must change its selection from $\varphi(c^*)$ to some $\varphi(\widetilde{c})$ where $\widetilde{c} > c^*$, and we conclude (again) that the high-quality firm advertises less than the low-quality firm: $A_H = A(\widetilde{c}) < A_M(L) = A_L$.[14]

Proposition 5. *Under the generalized informative view of advertising, the high-quality firm advertises less than the low-quality firm in the least-cost separating equilibrium outcome.*

4.3. A memory-activation interpretation of informative advertising

To arrive at a generalized informative view of advertising, what we have added beyond the base case is that the ability of advertising to bring in more consumers may be lower when consumers have more optimistic beliefs, holding fixed the price (i.e., $G_{Ab} \leq 0$). In other words, advertising shifts out demand at a rate that depends on consumer beliefs, with the shift perhaps being greatest when consumers are least optimistic. This is consistent with the idea that *large doses* of advertising and *optimistic* beliefs both work to increase patronage and are *substitutes*. Intuitively, if beliefs are pessimistic, advertising can be very effective; however, if beliefs are optimistic, then there may be less that is left for advertising to do.

[14] While we emphasize the relationship between advertising and quality, we also have results that pertain to the price-quality relationship. In particular, we can establish the following: Under the generalized informative view of advertising, if $d_{pb} \geq 0$ and $\pi_{pp}(p, A; 0, L) < 0$, the high-quality firm sets a higher price than the low-quality firm in the least-cost separating equilibrium outcome. Details are available from the authors.

An example will illustrate this idea. Let us suppose that all potential customers are aware of the general advertising expenditures and pricing selected by the firm. They are all informed in this sense, and they have all formed general beliefs, b, about the product. The issue is rather whether, at a given purchase moment in time, the product is in the consumer's "consideration set". Suppose that there are *two* ways that a product can enter into a consumer's consideration set. *First*, it may be that the consumer actually receives a particular ad and is, thus, prompted to recall the product and consider a potential purchase. This is a *memory-activation* role for advertising, although one that presumes no direct prior experience with the product. Following Butters (1977), let us assume that the probability that no specific ad reaches a given consumer is $e^{-A/N}$, when A messages are sent randomly to N consumers. Thus, specific ads are received at the purchase moment by $N(1-e^{-A/N})$ consumers. *Second*, absent any direct exposure to an advertising message, it may be that a consumer is more likely to consider a product for purchase at a given moment in time, when the consumer has assigned a high belief to the quality of the product. In particular, if, after learning the firm's general price and advertising behavior, the consumer assigns a belief b as to the likelihood that the product is of high quality, then the consumer is more likely to consider the product for potential purchase at a given moment in time when b is larger. The assumption here is that perceived higher-quality products are more likely to be at the forefront of the consumer's consciousness. Let $f(b)$ denote the probability that the consumer does not consider the product for possible purchase; thus, we assume that $f(b)$ is decreasing. The number of consumers for whom the product springs to mind (independently of advertising) is thus $N(1-f(b))$.

To summarize, for this example we have that

$$\begin{aligned} G(A;b) &= N(1-e^{-A/N}) + N(1-f(b))e^{-A/N} \\ &= N(1-f(b)e^{-A/N}). \end{aligned}$$

Alternatively, $G(A;b)$ can be written as

$$G(A;b) = N(1-f(b))(1-e^{-A/N}) + Nf(b)(1-e^{-A/N}) + N(1-f(b))e^{-A/N}$$

from which we notice that the first term indicates a potential redundancy to advertising: some consumers are reached, for whom the product already springs to mind. In the end, everyone considers the product, except those that neither received a specific ad nor had the product come to mind. We observe that $G_A = f(b)e^{-A/N} > 0 > -f(b)e^{-A/N}/N = G_{AA}$ and $G_b = -Nf'(b)e^{-A/N} > 0$. Furthermore,

$$\frac{d}{db}\frac{G_A}{G} = \frac{f'(b)e^{-A/N}}{N(1-f(b)e^{-A/N})^2} < 0.$$

Thus, this example satisfies the assumptions made above.

It is interesting to compare this example with Nelson's (1974) perspective on the memory-activation role of advertising.[15] Nelson argues that advertising is more effective for high-quality goods, since the activation of consumer memory for such goods is more productive. Intuitively, advertising activates the recollection of a past purchase experience, and this recollection is more likely to result in a purchase for a high-quality product. Under this perspective, advertising and optimistic beliefs are *complements.* By contrast, in the example above, we do not allow for past purchase experiences (i.e., quality itself does not enter into expected consumer utility). Instead, the reception of an advertising message induces the consumer to recollect a prior belief about the product. Further, we assume that such a recollection may happen independently of advertising. In particular, when the belief formed is more optimistic, the recollection is more likely to occur anyhow. For this reason, an optimistic consumer is less likely to need advertising to recall his belief about the product.

This specific example offers a novel perspective on "informative advertising" in the context of memory activation. At a modeling level, the dilemma in developing such a perspective is to reconcile imperfect memories with rational (i.e., Bayesian) beliefs and signaling. Our approach is to separate the processes somewhat: all consumers see the aggregate advertising expenditures and the price and form Bayesian beliefs, but at the time of purchase only a fraction - which depends on aggregate advertising expenditures and beliefs - remember to consider the product.

5. The Persuasive View of Advertising

In the previous section we focused on informative advertising by positing a demand side in which the direct effect of increased advertising was to tap deeper into a given pool of potential customers with fixed willingness-to-pay for given assessments of the quality of the product on offer. In contrast, we now want to allow for the possibility that advertising directly shifts up the consumer valuations of the firm's product. We capture this by positing demand functions for which an increase in advertising results in an upward vertical shift of the demand function at all quantities; hence, we now assume that an increase in advertising generates an increase in willingness-to-pay at all quantities. These direct effects of advertising on demand can be understood as representing the *persuasive* view of advertising.[16]

We begin with some preliminaries and then turn to a base case, in which the magnitude of the upward shift in demand that is induced by advertising is independent of the beliefs held by consumers. We next analyze a generalized version of persuasive

[15] For further discussion of Nelson's (1974) work in this regard, see Bagwell (2005, Sections 2.3 and 6.2).

[16] It is also possible to understand this model as representing the *complementary* view of advertising. For further discussion of the persuasive and complementary views of advertising, see Bagwell (2005).

advertising that allows for interactions between advertising and beliefs. Thoughout the section, our focus is on establishing conditions under which the high-quality monopolist advertises less than would the low-quality monopolist, in the least-cost separating equilibrium outcome.

5.1. Preliminaries

We recall from Section 3 that, if a high-quality firm advertises *less* under complete information than does a low-quality monopolist, then the process of signaling will only reinforce this difference. On the other hand, if the high-quality firm advertises *more* under complete information than does a low-quality monopolist, then it is not *a priori* clear whether a high-quality firm continues to advertise more when signaling issues are introduced. Thus, a first step is to establish conditions under which the complete-information advertising levels of the high-quality and the low-quality firm can be ranked. Subsequently, we again consider the case in which complete-information advertising is greater for a high-quality firm and ask when this ranking is reversed by incomplete information and signaling.

Before proceeding with the formal analysis, we pause to develop some general intuition as to the conditions under which a high-quality firm is relatively attracted to a lower level of advertising. Recall that the firm's profit function is

$$\pi(p, A; b, q) = (p - c(q))D(p, A; b) - A,$$

where $c(H) > c(L)$. Consequently, the first-order condition for advertising is

$$\pi_A = (p - c(q))D_A(p, A; b) - 1 = 0.$$

Consider the complete-information setting. If, hypothetically, the mark-up were lower for a high-quality good, then D_A would have to be higher at the high-quality price-advertising selections with $b = 1$ than at the low-quality price-advertising selections with $b = 0$. Suppose, for example, that $D_{Ap} = D_{Ab} = 0$ and $D_{AA} < 0$. Note, in particular, that $D_{Ab} = 0$ implies that advertising and beliefs do not interact (at the margin) in affecting the willingness-to-pay of consumers. Hence, advertising and beliefs are *independent* "instruments" in this case. Given these assumptions, D_A can be made higher only by making A lower. From this line of reasoning we may discern an outline of conditions under which the complete-information advertising levels of the two types of the firm can be ranked. This requires some additional structure on the demand function (notably, the relationship between advertising intensity and quantity demanded), along with some conditions under which the complete-information mark-ups of the two types are rankable. Roughly, the latter is related to whether cost effects are more or less important for mark-ups than are belief effects. Finally, we note that assuming advertising and beliefs to be *substitutes* in affecting willingness-to-pay, that is, $D_{Ab} < 0$, should only help the argument: If higher beliefs (specifically, $b = 1$) work to lower D_A, then an even greater reduction in A is required from the high-quality monopolist in order to elevate D_A to the necessary level.

5.2. Persuasive advertising

5.2.1. The base case

Let us first specialize and assume that demand be given by

$$D(p, A; b) = f(A) + g(b) - p,$$

where $f > 0$, $f' > 0 > f''$, and $g(0) > c(H) > c(L) \geq 0$, $g'(b) > 0$. This demand specification, of course, subsumes $D_{Ap} = D_{Ab} = 0$. The base-case demand function is illustrated in Fig. 2 for different values of A and b, where higher A and/or b result in demands further from the origin.

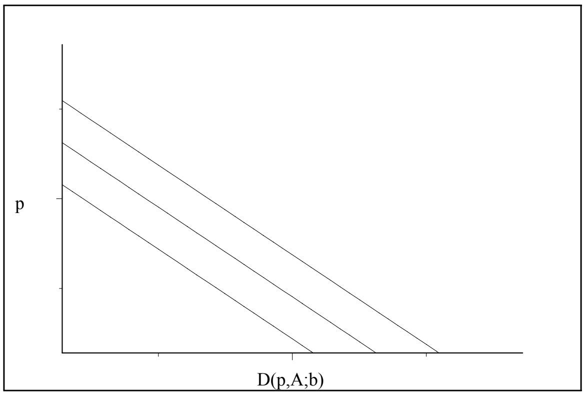

Fig. 2: Persuasive advertising - Base case

In the base case, we further assume that

$$\frac{d(ff')}{dA} = ff'' + (f')^2 \leq 0. \tag{5.1}$$

This simple demand function allows that both advertising and beliefs are (*independently*) capable of raising the willingness-to-pay of consumers, and it can be motivated with reference to some examples. Below, we will determine if additional assumptions are needed to ensure that these demand functions are sufficient to establish that we are in the regular case.

Example 1: Suppose that a mass of heterogenous consumers have unit demands, where a consumer of type θ elects to buy if $\theta + [bH + (1-b)L] + f(A) - p \geq 0$. Assuming that θ is distributed uniformly over the interval $[0,1]$, we find that the demand function is given as $D(p,A;b) = 1+[bH+(1-b)L]+f(A)-p$. Here, we may think of $g(b) = 1+[bH+(1-b)L]$, where the bracketed term is expected quality. If $1+L > c(H)$ and $H > L \geq 0$, then g satisfies the assumptions listed above. We may impose directly the remaining assumptions on f. As this example illustrates, a consumer's intrinsic valuation for a product (i.e., θ) is *enhanced* when the consumer becomes more optimistic about the quality of the product and/or when the consumer is exposed to advertising.

Example 2: Suppose directly that g satisfies the assumptions listed above and let $f(A) = A^{\alpha}$, where $\alpha \in (0, 1/2]$. Then it is straightforward to verify that the stated assumptions on f are all satisfied for $A > 0$. Likewise, the stated assumptions all hold when $f(A) = \ln(A+e)$, for $A \geq 0$. Our assumptions are thus satisfied by a large class of "reasonable" functions.

Formally, in the base case the profit function is

$$\pi(p, A; b, q) = (p - c(q))[f(A) + g(b) - p] - A.$$

Thus, the first-order conditions are[17]

$$\pi_p = f(A) + g(b) + c(q) - 2p = 0 \tag{5.2}$$

$$\pi_A = [p - c(q)]f'(A) - 1 = 0. \tag{5.3}$$

Using (5.2), we see that

$$p = \frac{f(A) + g(b) + c(q)}{2} \tag{5.4}$$

$$p - c(q) = \frac{f(A) + g(b) - c(q)}{2}. \tag{5.5}$$

We can combine (5.3) and (5.5) to obtain the following necessary condition on the profit-maximizing advertising level,

$$[f(A) + g(b) - c(q)]f'(A) = 2. \tag{5.6}$$

In particular, (5.6) must hold in the complete-information settings, in which $q = H$ and $b = 1$, and $q = L$ and $b = 0$, respectively. Thus, we have

$$[f(A_M(H)) + g(1) - c(H)]f'(A_M(H)) = 2, \tag{5.7}$$

[17] Given our assumptions, the second-order conditions are satisfied when the first-order conditions hold, since

$$\begin{aligned} \pi_{pp} &= -2 < 0 \\ \pi_{AA} &= [p - c(q)]f''(A) < 0 \\ | &\; J | \equiv \pi_{pp}\pi_{AA} - (\pi_{Ap})^2 = -[ff'' + (f')^2 + f''(g - c(q)] > 0 \end{aligned}$$

where J denotes the *Jacobian* matrix associated with the system of first-order conditions.

$$[f(A_M(L)) + g(0) - c(L)]f'(A_M(L)) = 2. \tag{5.8}$$

Subtracting (5.8) from (5.7), we thus obtain our key equation,

$$\begin{aligned} 0 \;=\; & f(A_M(H))f'(A_M(H)) - f(A_M(L))f'(A_M(L)) \\ & +[g(1) - c(H)]f'(A_M(H)) - [g(0) - c(L)]f'(A_M(L)). \end{aligned} \tag{5.9}$$

We now distinguish between two cases.

Case 1 (Cost effects dominate belief effects): $g(1) - c(H) < g(0) - c(L)$

Suppose that $A_M(H) \geq A_M(L)$. Under (5.1), this implies that

$$f(A_M(H))f'(A_M(H)) - f(A_M(L))f'(A_M(L)) \leq 0. \tag{5.10}$$

We may now derive that

$$\begin{aligned} 0 &\leq [g(1) - c(H)]f'(A_M(H)) - [g(0) - c(L)]f'(A_M(L)) \\ &< [g(0) - c(L)]f'(A_M(H)) - [g(0) - c(L)]f'(A_M(L)) \\ &= [g(0) - c(L)][f'(A_M(H)) - f'(A_M(L))] \\ &\leq 0, \end{aligned}$$

where the first inequality uses (5.9) and (5.10), the second inequality uses the assumption that cost effects dominate belief effects along with $f' > 0$, and the final inequality uses our supposition that $A_M(H) \geq A_M(L)$ along with our assumptions that $f'' < 0 < f'$ and $g(0) - c(L) > 0$. Having reached a contradiction, we conclude that the supposition is incompatible with our other assumptions. Thus, when cost effects dominate, it must be true that under complete information, the high-quality firm advertises *less* than does the low-quality firm, that is, $A_M(H) < A_M(L)$.

Case 2 (Belief effects dominate cost effects): $g(0) - c(L) < g(1) - c(H)$

Suppose $A_M(L) \geq A_M(H)$. Under (5.1), this implies that

$$f(A_M(H))f'(A_M(H)) - f(A_M(L))f'(A_M(L)) \geq 0. \tag{5.11}$$

We may now derive that

$$\begin{aligned} 0 &\geq [g(1) - c(H)]f'(A_M(H)) - [g(0) - c(L)]f'(A_M(L)) \\ &> [g(0) - c(L)]f'(A_M(H)) - [g(0) - c(L)]f'(A_M(L)) \\ &= [g(0) - c(L)][f'(A_M(H)) - f'(A_M(L))] \\ &\geq 0, \end{aligned}$$

where the first inequality uses (5.9) and (5.11), the second inequality uses the assumption that belief effects dominate cost effects along with $f' > 0$, and the final inequality uses our supposition that $A_M(L) \geq A_M(H)$ along with our assumptions that $f'' < 0 < f'$ and $g(0)-c(L) > 0$. Having, again, reached a contradiction, we conclude that the supposition is incompatible with our other assumptions. Thus, when belief effects dominate, it must be true that under complete information, the high-quality firm advertises *more* than does the low-quality firm, $A_M(H) > A_M(L)$.

We summarize our analysis of the complete-information case in the following lemma.

Lemma 1. *For the base-case demand function,*

(i) if cost effects dominate belief effects, $g(1) - c(H) < g(0) - c(L)$, then under complete information the high-quality firm advertises less than the low-quality firm, $A_M(H) < A_M(L)$, and

(ii) if belief effects dominate cost effects, $g(0) - c(L) < g(1) - c(H)$, then under complete information the high-quality firm advertises more than the low-quality firm, $A_M(H) > A_M(L)$.

Comparison of the equilibrium advertising levels Let us now turn to the incomplete-information case. As noted previously, if the base-case demand function puts us in the regular case, so that a high-quality firm would advertise less and price higher if its unit costs were to increase, and if cost effects dominate belief effects, so that $A_M(H) < A_M(L)$, then we may immediately conclude that the high-quality firm advertises less in the least-cost separating equilibrium than does the low-quality firm, since $A_H < A_M(H) < A_M(L) = A_L$. Thus, we just need to check whether we are indeed in the regular case. Totally differentiating the first-order conditions, (5.2) and (5.3), when $b = 1$ and $c(q)$ is replaced with an arbitrary value c, we obtain that the solution $(p(c), A(c))$ satisfies

$$
\begin{aligned}
A'(c) &= \frac{-f'(A(c))}{\mid J(c) \mid} \\
p'(c) &= -\{\frac{f''(A(c))f(A(c)) + 2(f'(A(c))^2 + (g(1)-c)f''(A(c))}{2 \mid J(c) \mid}\},
\end{aligned}
$$

where $J(c)$ is the *Jacobian* matrix associated with the first-order conditions and $\mid J(c) \mid > 0$ under our maintained assumption that $\widehat{\pi}(p, A; c)$ has a unique, interior maximizer. We thus see that $A(c)$ is indeed decreasing. We observe that $p(c)$ is increasing if our assumptions are strengthened somewhat so that $g(1) > c$ and $f''f + 2(f')^2 \leq 0$. In any case, we may now conclude as follows.

Proposition 6. *For the base-case demand function, if cost effects dominate belief effects, $g(1) - c(H) < g(0) - c(L)$, then the high-quality firm advertises less than the low-quality firm in the least-cost separating equilibrium outcome.*

Finally, let us focus on the more interesting case in which belief effects dominate cost effects, $g(0) - c(L) < g(1) - c(H)$. Recall that, in this case, under complete information the high-quality firm advertises more than the low-quality firm, $A_M(H) > A_M(L)$. It is thus as yet unclear whether the high-quality monopolist advertises less in the refined separating equilibrium that emerges under incomplete information.

To make headway on this issue, we proceed as in Section 4 and compare the low-quality firm's selection, $(p_M(L), A_M(L))$, with the selection that a high-quality firm would make were its costs c^*, where c^* is defined by $A_M(L) = A(c^*)$. We establish above that $A(c)$ is decreasing. Our focus on the case wherein belief effects dominate cost effects ensures that $A_M(L) < A_M(H)$ and, thus, that $c^* > c(H) > c(L)$. If we find that the low-quality firm would mimic $(p(c^*), A(c^*))$, then we conclude that the high-quality firm achieves least-cost separation by making the selection that would maximize its complete-information profit were its costs $\widetilde{c} > c^*$. In this event, $A_H = A(\widetilde{c}) < A(c^*) = A_M(L) = A_L$, and, thus, we conclude that the high-quality firm advertises less than the low-quality firm in the least-cost separating equilibrium outcome.

This discussion motivates the following derivations. First, consider the profit difference from the perspective of the low-quality firm:

$$\begin{aligned} & \pi(p(c^*), A(c^*); 1, L) - \pi(p_M(L), A_M(L); 0, L) \\ = \ & [p(c^*) - c(L)]D(p(c^*), A(c^*); 1) - [p_M(L) - c(L)]D(p_M(L), A(c^*); 0) \\ > \ & [p(c^*) - c^*]D(p(c^*), A(c^*); 1) - [p_M(L) - c(L)]D(p_M(L), A(c^*); 0), \end{aligned}$$

where the equality uses $A(c^*) = A_M(L)$, while the inequality uses $c^* > c(L)$. Now, we can use the first-order conditions for advertising (i.e., $[p(c^*) - c^*]D_A(p(c^*), A(c^*); 1) = 1$ and $[p_M(L) - c(L)]D_A(p_M(L), A(c^*); 0) = 1$) and $D_A(p(c^*), A(c^*); 1) = D_A(p_M(L), A(c^*); 0) = f'(A(c^*))$ to obtain

$$\begin{aligned} & [p(c^*) - c^*]D(p(c^*), A(c^*); 1) - [p_M(L) - c(L)]D(p_M(L), A_M(L); 0) \\ = \ & \frac{D(p(c^*), A(c^*); 1)}{D_A(p(c^*), A(c^*); 1)} - \frac{D(p_M(L), A(c^*); 0)}{D_A(p_M(L), A(c^*); 0)} \\ = \ & \frac{D(p(c^*), A(c^*); 1) - D(p_M(L), A(c^*); 0)}{f'(A(c^*))}. \end{aligned}$$

Further, inserting the solutions for prices (i.e., $p(c^*) = [f(A(c^*)) + g(1) + c^*]/2$ and $p_M(L) = [f(A(c^*)) + g(0) + c(L)]/2$) and demands (i.e., $D(p(c^*), A(c^*); 1) = f(A(c^*)) + g(1) - p(c^*)$ and $D(p_M(L), A(c^*); 0) = f(A(c^*)) + g(0) - p_M(L)$), we obtain

$$
\begin{aligned}
& \frac{D(p(c^*), A(c^*); 1) - D(p_M(L), A(c^*); 0)}{f'(A(c^*))} \\
= \; & \frac{[g(1) - g(0)] - [p(c^*) - p_M(L)]}{f'(A(c^*))} \\
= \; & \frac{[g(1) - c^*] - [g(0) - c(L)]}{2f'(A(c^*))}.
\end{aligned}
$$

Hence, we may now conclude that

$$\pi(p(c^*), A(c^*); 1, L) - \pi(p_M(L), A_M(L); 0, L) > \frac{[g(1) - c^*] - [g(0) - c(L)]}{2f'(A(c^*))}$$

if belief effects dominate cost effects. However, recall from (5.8) that

$$[f(A(c^*)) + g(0) - c(L)]f'(A(c^*)) = 2.$$

Analogously, the first-order conditions for $p(c^*)$ and $A(c^*)$ can be combined to yield

$$[f(A(c^*)) + g(1) - c^*]f'(A(c^*)) = 2.$$

Combining and using $f'(A(c^*)) > 0$, this is equivalent to

$$g(1) - c^* = g(0) - c(L),$$

and we finally conclude that

$$\pi(p(c^*), A(c^*); 1, L) - \pi(p_M(L), A_M(L); 0, L) > 0$$

if $A(c^*) = A_M(L)$. Thus, we can state the following result.

Proposition 7. *Consider the base-case demand function and suppose that belief effects dominate cost effects, $g(0) - c(L) < g(1) - c(H)$. Define $c^* > c(H)$ by $A(c^*) = A_M(L)$. Then $c^* = c(L) + (g(1) - g(0))$, and it follows that the high-quality firm advertises less than the low-quality firm in the least-cost separating equilibrium outcome.*

Combining Propositions 6 and 7, we find that the high-quality firm always advertises less than the low-quality firm, when persuasive advertising is of the form that gives rise to our base-case demand function.[18] This parallels our findings for informative advertising in Section 4. Summarizing:

Proposition 8. *Under the persuasive view of advertising, the high-quality firm advertises less than the low-quality firm in the least-cost separating equilibrium outcome.*

[18] If belief effects and cost effects exactly offset, $g(0) - c(L) = g(1) - c(H)$, then $A_M(H) = A_M(L)$, and hence $A_H = A(\tilde{c}) < A_M(H) = A_M(L) = A_L$. Thus, our finding holds in this remaining case as well.

5.2.2. Generalized version of persuasive advertising

We now consider the following generalized representation of persuasive advertising

$$D(p, A; b) = h(A; b) - p,$$

where $h_A > 0 > h_{AA}$, $h > c(H) > c(L) \geq 0$, and $h_b > 0$. We assume further that

$$\frac{d(h - c(q))h_A}{dA} = (h - c(q))h_{AA} + (h_A)^2 < 0. \tag{5.12}$$

Again, the demand function allows that advertising and beliefs both raise the willingness-to-pay of consumers, while we have not yet specified the possible interactions between advertising and beliefs in the function h.

The profit function is now

$$\pi(P, A; b, q) = (p - c(q))[h(A; b) - p] - A,$$

and the first-order conditions are[19]

$$\pi_p = h(A; b) + c(q) - 2p = 0 \tag{5.13}$$

$$\pi_A = [p - c(q)]h_A(A; b) - 1 = 0. \tag{5.14}$$

Using (5.13), we find that

$$p = \frac{h(A; b) + c(q)}{2} \tag{5.15}$$

$$p - c(q) = \frac{h(A; b) - c(q)}{2}. \tag{5.16}$$

We may combine (5.14) and (5.16) to obtain the following necessary condition on the profit-maximizing advertising level,

$$[h(A; b) - c(q)]h_A(A; b) = 2. \tag{5.17}$$

In particular, (5.17) must hold in the complete-information settings, in which $q = H$ and $b = 1$, and $q = L$ and $b = 0$, respectively. Thus, we have that

$$[h(A_M(H); 1) - c(H)]h_A(A_M(H); 1) = 2 \tag{5.18}$$

[19] Under our assumptions, the second-order conditions are satisfied when the first-order conditions hold, since

$$\begin{aligned} \pi_{pp} &= -2 < 0 \\ \pi_{AA} &= [p - c(q)]h_{AA}(A; b) < 0 \\ &| \quad J \,|\equiv \pi_{pp}\pi_{AA} - (\pi_{Ap})^2 = -[(h - c(q))h_{AA} + (h_A)^2] > 0. \end{aligned}$$

$$[h(A_M(L);0) - c(L)]h_A(A_M(L);0) = 2. \tag{5.19}$$

Subtracting (5.19) from (5.18), we thus obtain a new version of the key equation,

$$0 = [h(A_M(H);1) - c(H)]h_A(A_M(H);1) - [h(A_M(L);0) - c(L)]h_A(A_M(L);0). \tag{5.20}$$

For consistency with the approach for the base case, we initially distinguish between two cases where cost effects and belief effects dominate, respectively. By adding the interactions $D_{Ab} \neq 0$, we note in advance that the cases are no longer exhaustive.

Case 1 (Cost effects dominate belief effects, *and* advertising and beliefs are substitutes): At any given A, $h(A;1) - c(H) < h(A;0) - c(L)$, and $h_{Ab} \leq 0$.

In this case, we have that

$$\begin{aligned} 0 &> [h(A;1) - c(H)]h_A(A;0) - [h(A;0) - c(L)]h_A(A;0) \\ &\geq [h(A;1) - c(H)]h_A(A;1) - [h(A;0) - c(L)]h_A(A;0), \end{aligned}$$

where the first inequality follows from $h(A;1) - c(H) < h(A;0) - c(L)$, while the second inequality is a consequence of $h_{Ab} \leq 0$. In particular, in *Case 1* when $A = A_M(L)$, we have

$$0 > [h(A_M(L);1) - c(H)]h_A(A_M(L);1) - [h(A_M(L);0) - c(L)]h_A(A_M(L);0). \tag{5.21}$$

Let us now suppose that $A_M(H) \geq A_M(L)$. Using (5.12) and (5.21), we conclude that

$$0 > [h(A_M(H);1) - c(H)]h_A(A_M(H);1) - [h(A_M(L);0) - c(L)]h_A(A_M(L);0). \tag{5.22}$$

But (5.22) contradicts our key equation (5.20), and so we conclude that the supposition that $A_M(H) \geq A_M(L)$ is incompatible with our assumptions. It thus follows that $A_M(H) < A_M(L)$.

Case 2 (Belief effects dominate cost effects, *and* advertising and beliefs are complements): At any given A, $h(A;1) - c(H) > h(A;0) - c(L)$, and $h_{Ab} \geq 0$.

In this case, we have that

$$\begin{aligned} 0 &< [h(A;1) - c(H)]h_A(A;1) - [h(A;0) - c(L)]h_A(A;1) \\ &\leq [h(A;1) - c(H)]h_A(A;1) - [h(A;0) - c(L)]h_A(A;0), \end{aligned}$$

where the first inequality follows from $h(A;1) - c(H) > h(A;0) - c(L)$, while the second inequality is a consequence of $h_{Ab} \geq 0$. In particular, in *Case 2* when $A = A_M(H)$, we have

$$0 < [h(A_M(H);1) - c(H)]h_A(A_M(H);1) - [h(A_M(H);0) - c(L)]h_A(A_M(H);0). \tag{5.23}$$

Let us now suppose that $A_M(L) \geq A_M(H)$. Using (5.12) and (5.23), we conclude that

$$0 < [h(A_M(H);1) - c(H)]h_A(A_M(H);1) - [h(A_M(L);0) - c(L)]h_A(A_M(L);0). \quad (5.24)$$

But (5.23) contradicts our key equation (5.20), and so we conclude that our supposition that $A_M(L) \geq A_M(H)$ is incompatible with our assumptions. It thus follows that $A_M(L) < A_M(H)$.

We summarize results so far in the following lemma.

Lemma 2. *For the demand function described by our generalized version of persuasive advertising,*

(i) if cost effects dominate belief effects, $h(A;1) - c(H) < h(A;0) - c(L)$, and advertising and beliefs are substitutes, $h_{Ab} \leq 0$, then under complete information the high-quality firm advertises less than the low-quality firm, that is, $A_M(H) < A_M(L)$, and

(ii) if belief effects dominate cost effects, $h(A;1) - c(H) > h(A;0) - c(L)$, and advertising and beliefs are complements, $h_{Ab} \geq 0$, then under complete information the high-quality firm advertises more than the low-quality firm, that is, $A_M(H) > A_M(L)$.

We reiterate that this characterization of cases under the generalized version is not exhaustive, since each case involves assumptions that must hold for all advertising levels, A. In the special case that $h(A;b) = f(A)+g(b)$, however, we have that $[h(A;1)-c(H)]-[h(A;0) - c(L)] = [g(1) - c(H)] - [g(0) - c(L)]$ is independent of A and that $h_{Ab} = 0$. Then, the two cases are exhaustive, and this special case, indeed, corresponds to our base case. To see the relation between the assumptions, notice that our assumption,

$$\frac{d(h - c(q))h_A}{dA} = (h - c(q))h_{AA} + (h_A)^2 < 0,$$

in this case can be rewritten as

$$(f + g - c(q))f'' + (f')^2 = ff'' + (f')^2 + (g - c(q))f'' \leq 0.$$

In our base case model, we achieved this inequality with the separate assumptions that $ff'' + (f')^2 \leq 0$, $g > c(q)$ and $f'' < 0$.

Heuristically and somewhat loosely (by continuity), the import of Lemma 2 is the following. If cost effects are strong compared to belief effects *and* advertising and beliefs are either substitutes or weak complements, then under complete information the high-quality firm advertises *less* than the low-quality firm. In contrast, if beliefs effects are strong compared to costs effects *and* advertising and beliefs are either complements or weak substitutes, then under complete information the high-quality firm advertises *more* than the low-quality firm. A full characterization for the complete-information case would require further specification of h. However, below we show that whether cost effects or beliefs effects dominate, and, thus, whether the high-quality firm advertises more or less than the low-quality firm under complete information, if advertising and beliefs are *substitutes*, $D_{Ab} \leq 0$, then the high-quality firm advertises less than the low-quality firm in the refined equilibrium of the game with incomplete information.

Comparison of the equilibrium advertising levels As discussed previously, if the generalized-version demand function puts us in the regular case, so that a high-quality firm would advertise less and price higher if its unit costs were to increase, *and* if cost effects dominate belief effects and advertising and beliefs are substitutes, so that $A_M(H) < A_M(L)$, then we conclude that the high-quality firm advertises less in the least-cost separating equilibrium than the low-quality firm, since $A_H < A_M(H) < A_M(L) = A_L$. We therefore investigate whether we are, indeed, in the regular case. Totally differentiating the first-order conditions, (5.13) and (5.14), when $b = 1$ and $c(q)$ is replaced with an aribtrary value c, we obtain that the solution $(p(c), A(c))$ satisfies

$$\begin{aligned} A'(c) &= \frac{-h_A(A(c);1)}{\mid J(c) \mid} \\ p'(c) &= -\{\frac{(h(A(c);1) - c)h_{AA}(A(c);1) + 2(h_A(A(c);1))^2}{2 \mid J(c) \mid}\}, \end{aligned}$$

where $\mid J(c) \mid > 0$ is again the determinant of the associated Jacobian matrix. We thus see that $A(c)$ is indeed decreasing. We also observe that $p(c)$ is increasing if our assumptions are strengthened somewhat so that $(h(A(c);1)-c)h_{AA}(A(c);1)+2(h_A(A(c);1))^2 < 0$. In any case, we may now conclude as follows.

Proposition 9. *For the demand function described by the generalized version of persuasive advertising, if cost effects dominate belief effects, $h(A;1)-c(H) < h(A;0)-c(L)$, advertising and beliefs are substitutes, $h_{Ab} \leq 0$, then the high-quality firm advertises less than the low-quality firm in the least-cost separating equilibrium outcome.*

The proposition describes a set of sufficient conditions for $A_M(H) < A_M(L)$ and thereby for the conclusion that $A_H < A_L$. More generally, in light of our finding that $A'(c) < 0$, we may conclude that $A_H < A_L$ whenever $A_M(H) \leq A_M(L)$.

We next turn to the case where $A_M(H) > A_M(L)$. As noted above, this case can be achieved while maintaining our assumption that advertising and beliefs are substitutes, if belief effects are strong compared to cost effects. Let us therefore suppose that $A_M(H) > A_M(L)$, $h_{Ab} \leq 0$ and $h(A;1) - c(H) > h(A;0) - c(L)$. As in the base case, we hypothetically compare the low-quality firm's selection, $(p_M(L), A_M(L))$, with the selection that a high-quality firm would make were its costs c^*, where c^* is defined by

$A_M(L) = A(c^*)$. Given that $A'(c) < 0$ and $A_M(H) > A_M(L) = A(c^*)$, it follows that $c^* > c(H) > c(L)$.

Taking the perspective of the low-quality firm, we may use $c^* > c(L)$ to establish

$$\begin{aligned} & \pi(p(c^*), A(c^*); 1, L) - \pi(p_M(L), A_M(L); 0, L) \\ = \; & [p(c^*) - c(L)]D(p(c^*), A(c^*); 1) - [p_M(L) - c(L)]D(p_M(L), A(c^*); 0) \\ > \; & [p(c^*) - c^*]D(p(c^*), A(c^*); 1) - [p_M(L) - c(L)]D(p_M(L), A(c^*); 0). \end{aligned}$$

Using the first-order conditions for advertising (i.e., $[p(c^*) - c^*]D_A(p(c^*), A(c^*); 1) = 1$ and $[p_M(L) - c(L)]D_A(p_M(L), A(c^*); 0) = 1$), $D_A(p(c^*), A(c^*); 1) = h_A(A(c^*); 1)$ and $D_A(p_M(L),\ A(c^*); 0) = h_A(A(c^*); 0)$ we get

$$\begin{aligned} & [p(c^*) - c^*]D(p(c^*), A(c^*); 1) - [p_M(L) - c(L)]D(p_M(L), A_M(L); 0) \\ = \; & \frac{D(p(c^*), A(c^*); 1)}{D_A(p(c^*), A(c^*); 1)} - \frac{D(p_M(L), A(c^*); 0)}{D_A(p_M(L), A(c^*); 0)} \\ = \; & \frac{D(p(c^*), A(c^*); 1)}{h_A(A(c^*); 1)} - \frac{D(p_M(L), A(c^*); 0)}{h_A(A(c^*); 0)}. \end{aligned}$$

Further, inserting the solutions for prices (i.e., $p(c^*) = [h(A(c^*); 1) + c^*]/2$ and $p_M(L) = [h(A(c^*); 0) + c(L)]/2$) and demands (i.e., $D(p(c^*), A(c^*); 1) = h(A(c^*); 1) - p(c^*)$ and $D(p_M(L), A(c^*); 0) = h(A(c^*); 0) - p_M(L)$), we obtain

$$\begin{aligned} & \frac{D(p(c^*), A(c^*); 1)}{h_A(A(c^*); 1)} - \frac{D(p_M(L), A(c^*); 0)}{h_A(A(c^*); 0)} \\ = \; & \frac{h(A(c^*); 1) - p(c^*)}{h_A(A(c^*); 1)} - \frac{h(A(c^*); 0) - p_M(L)}{h_A(A(c^*); 0)} \\ = \; & \frac{h(A(c^*); 1) - c^*}{2h_A(A(c^*); 1)} - \frac{h(A(c^*); 0) - c(L)}{2h_A(A(c^*); 0)}. \end{aligned}$$

Hence, we may now conclude that

$$\pi(p(c^*), A(c^*); 1, L) - \pi(p_M(L), A_M(L); 0, L) > \frac{h(A(c^*); 1) - c^*}{2h_A(A(c^*); 1)} - \frac{h(A(c^*); 0) - c(L)}{2h_A(A(c^*); 0)}$$

if $A_M(H) > A_M(L)$ so that $c^* > c(H) > c(L)$.

However, recall from (5.19) that

$$[h(A(c^*); 0) - c(L)]h_A(A(c^*); 0) = 2.$$

Likewise, the first-order conditions for $p(c^*)$ and $A(c^*)$ may be combined to yield

$$[h(A(c^*); 1) - c^*]h_A(A(c^*); 1) = 2.$$

Combining and using $h_A > 0$, this is equivalent to

$$h(A(c^*;1) - c^* = [h(A(c^*);0) - c(L)]\frac{h_A(A(c^*);0)}{h_A(A(c^*);1)}.$$

Hence, if $A(c^*) = A_M(L)$, then

$$\begin{aligned} & \pi(p(c^*), A(c^*); 1, L) - \pi(p_M(L), A_M(L); 0, L) \\ > \; & \frac{[h(A(c^*);0) - c(L)]h_A(A(c^*);0)}{2}\left[\frac{1}{[h_A(A(c^*);1)]^2} - \frac{1}{[h_A(A(c^*);0)]^2}\right] \\ = \; & \frac{1}{[h_A(A(c^*);1)]^2} - \frac{1}{[h_A(A(c^*);0)]^2} \\ \geq \; & 0, \end{aligned}$$

where the equality uses $[h(A(c^*);0) - c(L)]h_A(A(c^*);0) = 2$, and the last inequality follows from $h_{Ab} \leq 0$.

We may now state the following result.

Proposition 10. *For the demand function described by the generalized version of persuasive advertising, if belief effects dominate cost effects, $h(A;1)-c(H) > h(A;0)-c(L)$, and if advertising and beliefs are substitutes, $h_{Ab} \leq 0$, then the high-quality firm advertises less than the low-quality firm in the least-cost separating equilibrium outcome.*

As explained previously, the argument is immediate if $A_M(H) \leq A_M(L)$. If instead $A_M(H) > A_M(L)$ obtains, then the argument presented immediately above establishes that $A_H = A(\widetilde{c}) < A_M(H) \leq A_M(L) = A_L$.

Combining the findings reported above, we have our final result on persuasive advertising.[20]

Proposition 11. *Under the generalized persuasive view of advertising, the high-quality firm advertises less than the low-quality firm in the least-cost separating equilibrium outcome if advertising and beliefs are substitutes.*

This last result formalizes our opening statement that *substitutability* of advertising and beliefs in affecting willingness-to-pay strengthens whatever tendency there might be of the high-quality firm to lower advertising below the level of the low-quality firm in the refined equilibrium.

[20] If $A_M(H) \leq A_M(L)$, the argument is immediate, and so we addressed in detail the case in which $A_M(H) > A_M(L)$. By Lemma 2, it then must be that either belief effects dominate cost effects or neither effect dominates the other for all advertising levels. In any event, the proof that precedes Proposition 10 utilizes only the assumptions that $A_M(H) > A_M(L)$ and $h_{Ab} \leq 0$. Proposition 11 is thus established.

6. Concluding Remarks

An extensive literature considers the role of advertising as a signal of quality. We contribute to this literature by showing that, for a wide range of demand functions, in the least-cost separating equilibrium, a high-quality monopolist advertises *less* than would a low-quality monopolist. We develop this result in a general static signaling model that allows for price and advertising to serve as joint signals of quality. As discussed in the Introduction, the price and advertising selections that emerge in the static signaling model can be understood to correspond to the choices that would be made in the introductory phase of a two-phase signaling framework. Our work thus suggests that the correlation between advertising and quality will be *negative* in the introductory phase of a product's life cycle. The correlation should "strengthen" as the product enters its mature phase, as the correlation will then be less negative or even positive. Consistent with earlier work, we also predict that the price-quality correlation weakens over time. These predictions are broadly consistent with some recent empirical findings and may serve a useful role by guiding the specification of future empirical analyses.

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Appendix

In this Appendix, we first prove Proposition 1 and 2, and then we rule out pooling outcomes.

Proof of Proposition 1

We want to show that some pair (p, A) on $\varphi(c)$ uniquely maximizes $\pi(p, A; 1, H)$ given the binding constraint $\pi(p, A; 1, L) = \Pi_M(L)$.

Our first step is to show that $\pi(\varphi(c); 1, H)$ is decreasing in c for $c > c(H)$ and increasing in c for $c < c(H)$. Take any pair (c', c'') such that $c' < c''$. Then

$$(p(c') - c')D(p(c'), A(c'); 1) - A(c') - (p(c'') - c')D(p(c''), A(c''); 1) + A(c'') > 0 \quad \text{(A1)}$$

and

$$(p(c'') - c'')D(p(c''), A(c''); 1) - A(c'') - (p(c') - c'')D(p(c'), A(c'); 1) + A(c') > 0 \quad \text{(A2)}$$

Adding (A2) and (A2) we get

$$(c'' - c')(D(p(c'), A(c'); 1) - D(p(c''), A(c''); 1)) > 0$$

and it follows that

$$D(p(c'), A(c'); 1) > D(p(c''), A(c''); 1) \quad \text{(A3)}$$

Using (A1) we evaluate

$$\begin{aligned}
& \pi(\varphi(c'); 1, H) - \pi(\varphi(c''); 1, H) \\
= \; & (p(c') - c(H))D(p(c'), A(c'); 1) - A(c') - (p(c'') - c(H))D(p(c''), A(c''); 1) + A(c'') \\
> \; & (p(c') - c(H))D(p(c'), A(c'); 1) - A(c') - (p(c'') - c(H))D(p(c''), A(c''); 1) + A(c'') \\
& -[(p(c') - c')D(p(c'), A(c'); 1) - A(c') - (p(c'') - c')D(p(c''), A(c''); 1) + A(c'')] \\
= \; & (c' - c(H))(D(p(c'), A(c'); 1) - D(p(c''), A(c''); 1))
\end{aligned}$$

But, using (A3), we conclude that $\pi(\varphi(c'); 1, H) > \pi(\varphi(c''); 1, H)$ when $c(H) < c' < c''$, while $\pi(\varphi(c'); 1, H) < \pi(\varphi(c''); 1, H)$ when $c' < c'' < c(H)$. This proves our claim.

An exactly analogous argument, which we leave to the reader, establishes that $\pi(\varphi(c); 1, L)$ is decreasing in c for $c > c(L)$ and increasing in c for $c < c(L)$.

Now, turn to the proposition. Take any (p, A) such that $\pi(p, A; 1, L) = \Pi_M(L)$ but which is *not* on the one-dimensional sub-manifold $\varphi(c)$. Then, using our assumed boundary conditions and the properties of $\pi(\varphi(c); 1, L)$ just established, there exists *one* $\widetilde{c} > c(L)$ such that $\pi(\varphi(\widetilde{c}); 1, L) = \Pi_M(L)$ and one $\widetilde{\widetilde{c}} < c(L)$ such that $\pi(\varphi(\widetilde{\widetilde{c}}); 1, L) = \Pi_M(L)$. Recalling (3.2) in the text,

$$\pi(p_M(H), A_M(H); 1, L) = \pi(\varphi(c(H)); 1, L) > \Pi_M(L)$$

it follows that $\widetilde{\widetilde{c}} < c(H) < \widetilde{c}$.

Further, by construction, we have

$$(p(\widetilde{c}) - \widetilde{c})D(p(\widetilde{c}), A(\widetilde{c}); 1) - A(\widetilde{c}) - (p - \widetilde{c})D(p, A; 1) + A > 0 \quad \text{(A4)}$$

$$(p(\widetilde{\widetilde{c}}) - \widetilde{\widetilde{c}})D(p(\widetilde{\widetilde{c}}), A(\widetilde{\widetilde{c}}); 1) - A(\widetilde{\widetilde{c}}) - (p - \widetilde{\widetilde{c}})D(p, A; 1) + A > 0 \quad \text{(A5)}$$

$$(p - c(L))D(p, A; 1) - A - (p(\widetilde{c}) - c(L))D(p(\widetilde{c}), A(\widetilde{c}); 1) + A(\widetilde{c}) = 0 \quad \text{(A6)}$$

and

$$(p - c(L))D(p, A; 1) - A - (p(\widetilde{\widetilde{c}}) - c(L))D(p(\widetilde{\widetilde{c}}), A(\widetilde{\widetilde{c}}); 1) + A(\widetilde{\widetilde{c}}) = 0 \quad \text{(A7)}$$

Adding (A4) and (A6), we obtain

$$(c(L) - \widetilde{c})(D(p(\widetilde{c}), A(\widetilde{c}); 1) - D(p, A; 1)) > 0$$

and, since $\widetilde{c} > c(L)$, we conclude that $D(p, A; 1) > D(p(\widetilde{c}), A(\widetilde{c}); 1)$. Similarly, adding (A5) and (A7), we obtain

$$(c(L) - \widetilde{\widetilde{c}})(D(p(\widetilde{\widetilde{c}}), A(\widetilde{\widetilde{c}}); 1) - D(p, A; 1)) > 0$$

and, since $\widetilde{\widetilde{c}} < c(L)$, we conclude that $D(p(\widetilde{\widetilde{c}}), A(\widetilde{\widetilde{c}}); 1) > D(p, A; 1)$. Hence,

$$D(p(\widetilde{\widetilde{c}}), A(\widetilde{\widetilde{c}}); 1) > D(p, A; 1) > D(p(\widetilde{c}), A(\widetilde{c}); 1) \quad \text{(A8)}$$

To eliminate any $(p, A) \notin \{\varphi(\widetilde{\widetilde{c}}), \varphi(\widetilde{c})\}$, we want to show that

$$\pi(p, A; 1, H) < \max\{\pi(\varphi(\widetilde{\widetilde{c}}); 1, H), \pi(\varphi(\widetilde{c}); 1, H)\}$$

Now,

$$\begin{aligned}
& \pi(\varphi(\widetilde{\widetilde{c}}); 1, H) - \pi(\varphi(\widetilde{c}); 1, H) \\
= \; & \pi(\varphi(\widetilde{\widetilde{c}}); 1, H) - \pi(\varphi(\widetilde{c}); 1, H) - [\pi(\varphi(\widetilde{\widetilde{c}}); 1, L) - \pi(\varphi(\widetilde{c}); 1, L)] \\
= \; & (p(\widetilde{\widetilde{c}}) - c(H))D(p(\widetilde{\widetilde{c}}), A(\widetilde{\widetilde{c}}); 1) - A(\widetilde{\widetilde{c}}) - (p(\widetilde{c}) - c(H))D(p(\widetilde{c}), A(\widetilde{c}); 1) + A(\widetilde{c}) \\
& -(p(\widetilde{\widetilde{c}}) - c(L))D(p(\widetilde{\widetilde{c}}), A(\widetilde{\widetilde{c}}); 1) + A(\widetilde{\widetilde{c}}) + (p(\widetilde{c}) - c(L))D(p(\widetilde{c}), A(\widetilde{c}); 1) - A(\widetilde{c}) \\
= \; & (c(L) - c(H))(D(p(\widetilde{\widetilde{c}}), A(\widetilde{\widetilde{c}}); 1) - D(p(\widetilde{c}), A(\widetilde{c}); 1))
\end{aligned}$$

By (A8) we have $D(p(\widetilde{\widetilde{c}}), A(\widetilde{\widetilde{c}}); 1) > D(p(\widetilde{c}), A(\widetilde{c}); 1)$, and it follows that

$$sign\{\pi(\varphi(\widetilde{\widetilde{c}}); 1, H) - \pi(\varphi(\widetilde{c}); 1, H)\} = sign\{c(L) - c(H)\}$$

We conclude that

$$\max\{\pi(\varphi(\widetilde{\widetilde{c}}); 1, H), \pi(\varphi(\widetilde{c}); 1, H)\} = \begin{cases} \pi(\varphi(\widetilde{\widetilde{c}}); 1, H) & if \quad c(L) > c(H) \\ \pi(\varphi(\widetilde{c}); 1, H) & if \quad c(L) < c(H) \end{cases}$$

Consider the two cases in turn.

a) $c(L) > c(H)$. Using (A7), we evaluate

$$\begin{aligned}
& \pi(\varphi(\widetilde{\widetilde{c}});1,H) - \pi(p,A;1,H) \\
= \; & \pi(\varphi(\widetilde{\widetilde{c}});1,H) - \pi(p,A;1,H) - [\pi(\varphi(\widetilde{\widetilde{c}});1,L) - \pi(p,A;1,L)] \\
= \; & (p(\widetilde{\widetilde{c}}) - c(H))D(p(\widetilde{\widetilde{c}}),A(\widetilde{\widetilde{c}});1) - A(\widetilde{\widetilde{c}}) - (p - c(H))D(p,A;1) + A \\
& -(p(\widetilde{\widetilde{c}}) - c(L))D(p(\widetilde{\widetilde{c}}),A(\widetilde{\widetilde{c}});1) + A(\widetilde{\widetilde{c}}) + (p - c(L))D(p,A;1) - A \\
= \; & (c(L) - c(H))(D(p(\widetilde{\widetilde{c}}),A(\widetilde{\widetilde{c}});1) - D(p,A;1)) \\
> \; & 0
\end{aligned}$$

where the inequality follows from (A8) and $c(L) > c(H)$. Hence, $\varphi(\widetilde{\widetilde{c}})$ maximizes $\pi(p',A';1,H)$ subject to $\pi(p',A';1,L) = \Pi_M(L)$.

b) $c(L) < c(H)$. Using (A6), we evaluate

$$\begin{aligned}
& \pi(\varphi(\widetilde{c});1,H) - \pi(p,A;1,H) \\
= \; & \pi(\varphi(\widetilde{c});1,H) - \pi(p,A;1,H) - [\pi(\varphi(\widetilde{c});1,L) - \pi(p,A;1,L)] \\
= \; & (p(\widetilde{c}) - c(H))D(p(\widetilde{c}),A(\widetilde{c});1) - A(\widetilde{c}) - (p - c(H))D(p,A;1) + A \\
& -(p(\widetilde{c}) - c(L))D(p(\widetilde{c}),A(\widetilde{c});1) + A(\widetilde{c}) + (p - c(L))D(p,A;1) - A \\
= \; & (c(L) - c(H))(D(p(\widetilde{c}),A(\widetilde{c});1) - D(p,A;1)) \\
> \; & 0
\end{aligned}$$

where the inequality follows from (A8) and $c(L) < c(H)$. Hence, $\varphi(\widetilde{c})$ maximizes $\pi(p',A';1,H)$ subject to $\pi(p',A';1,L) = \Pi_M(L)$.

Combining a) and b) completes the proof of Proposition 1. ■

Proof of Proposition 2

Proposition 2 follows immediately from the proof of Proposition 1. In the normal case $c(L) < c(H)$, and the solution to (PH) is at $\varphi(\widetilde{c})$ for some $\widetilde{c} > c(H)$. ■

Destabilization of pooling outcomes

We shall concentrate on pure strategy pooling equilibria, while noting that the arguments carry over to any hybrid equilibrium.

Fix some putative pooling equilibrium pair $(\widehat{p},\widehat{A})$, and suppose that both types of the firm play this pair with probability one. It follows that $b(\widehat{p},\widehat{A}) = b^0$, where $b^0 < 1$ denotes the prior probability assessment that quality is high. Again, we consider the two cases alluded to above in turn.

a) $c(L) > c(H)$. If $c(L) > c(H)$, then under our assumed boundary condition there exists some $c'' < c(L)$ such that

$$(\widehat{p} - c(L))D(\widehat{p},\widehat{A};b^0) - \widehat{A} - (p(c'') - c(L))D(p(c''),A(c'');1) + A(c'') = 0 \qquad \text{(A9)}$$

By construction, we also have

$$(p(c'') - c'')D(p(c''), A(c''); 1) - A(c'') - (\widehat{p} - c'')D(\widehat{p}, \widehat{A}; b^0) + \widehat{A} > 0 \tag{A10}$$

Add (A9) and (A10) to obtain

$$(c'' - c(L))(D(\widehat{p}, \widehat{A}; b^0) - D(p(c''), A(c''); 1)) > 0$$

which implies that $D(\widehat{p}, \widehat{A}; b^0) < D(p(c''), A(c''); 1)$.

Evaluate

$$(\widehat{p} - c(H))D(\widehat{p}, \widehat{A}; b^0) - \widehat{A} - (p(c'') - c(H))D(p(c''), A(c''); 1) + A(c'')$$

Subtract (A9) to obtain

$$(c(L) - c(H))(D(\widehat{p}, \widehat{A}; b^0) - D(p(c''), A(c''); 1)) < 0$$

We conclude that the high-quality firm strictly prefers $\varphi(c'')$ to the putative profile, provided that $b(\varphi(c'')) = 1$. By continuity, there exists some price-advertising pair close to $\varphi(c'')$ (e.g. $\varphi(c'' - \epsilon)$), which is strictly preferred by type H and strictly non-preferred by type L to the putative profile, if the associated beliefs are $b = 1$. Hence, under the Cho-Kreps (1987) refinement, the pooling profile is destabilized.

b) $c(L) < c(H)$. If $c(L) < c(H)$, then under our assumed boundary condition, there exists some $c' > c(L)$ such that

$$(\widehat{p} - c(L))D(\widehat{p}, \widehat{A}; b^0) - \widehat{A} - (p(c') - c(L))D(p(c'), A(c'); 1) + A(c') = 0 \tag{A11}$$

By construction, we also have

$$(p(c') - c')D(p(c'), A(c'); 1) - A(c') - (\widehat{p} - c')D(\widehat{p}, \widehat{A}; b^0) + \widehat{A} > 0 \tag{A12}$$

Add (A11) and (A12) to obtain

$$(c' - c(L))(D(\widehat{p}, \widehat{A}; b^0) - D(p(c'), A(c'); 1)) > 0$$

which implies that $D(\widehat{p}, \widehat{A}; b^0) > D(p(c'), A(c'); 1)$.

Evaluate

$$(\widehat{p} - c(H))D(\widehat{p}, \widehat{A}; b^0) - \widehat{A} - (p(c') - c(H))D(p(c'), A(c'); 1) + A(c')$$

Subtract (A11) to obtain

$$(c(L) - c(H))(D(\widehat{p}, \widehat{A}; b^0) - D(p(c'), A(c'); 1)) < 0$$

Again, we conclude that the high-quality firm strictly prefers $\varphi(c')$ to the putative profile, provided that $b(\varphi(c')) = 1$. Hence, under the Cho-Kreps (1987) refinement, the pooling profile is destabilized.

Combining a) and b) establishes that no pooling equilibria survive refinement. ■